

GCE Examinations
Advanced Subsidiary

Core Mathematics C4

Paper L

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



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C4 Paper L – Marking Guide

1.	<p>(a) $\frac{dn}{dt} = 0 \Rightarrow e^{0.5t} = 5$ $t = 2 \ln 5 = 3.219 \text{ mins} = 3 \text{ mins } 13 \text{ secs}$</p> <p>(b) $\int dn = \int (e^{0.5t} - 5) dt$ $n = 2e^{0.5t} - 5t + c$ $t = 0, n = 20 \Rightarrow 20 = 2 + c, \quad c = 18$ $n = 2e^{0.5t} - 5t + 18$</p> <p>(c) as t increases, n rapidly becomes very large \therefore not realistic</p>	M1 M1 A1 M1 A1 M1 A1 B1	(8)								
<hr/>											
2.	<p>$6x + y + x \frac{dy}{dx} - 4y \frac{dy}{dx} = 0$</p> <p>$(1, 4) \Rightarrow 6 + 4 + \frac{dy}{dx} - 16 \frac{dy}{dx} = 0, \quad \frac{dy}{dx} = \frac{2}{3}$</p> <p>grad of normal = $-\frac{3}{2}$</p> <p>$\therefore y - 4 = -\frac{3}{2}(x - 1)$ $2y - 8 = -3x + 3$ $3x + 2y - 11 = 0$</p>	M1 A2 M1 A1 M1 M1 A1	(8)								
<hr/>											
3.	<p>(a) $u = 2 - x^2 \Rightarrow \frac{du}{dx} = -2x$</p> <p>$I = \int \frac{1}{u} \times (-\frac{1}{2}) du = -\frac{1}{2} \int \frac{1}{u} du$ $= -\frac{1}{2} \ln u + c = -\frac{1}{2} \ln 2 - x^2 + c$</p> <p>(b) $= \int_0^{\frac{\pi}{4}} (\frac{1}{2} \sin 4x + \frac{1}{2} \sin 2x) dx$ $= [-\frac{1}{8} \cos 4x - \frac{1}{4} \cos 2x]_0^{\frac{\pi}{4}}$ $= (\frac{1}{8} - 0) - (-\frac{1}{8} - \frac{1}{4}) = \frac{1}{2}$</p>	M1 A1 M1 A1 M1 A1 M1 A1	(10)								
<hr/>											
4.	<p>(a) <table style="display: inline-table; border: none; vertical-align: middle;"> <tr> <td style="padding-right: 10px;">x</td> <td style="padding-right: 10px;">1</td> <td style="padding-right: 10px;">2</td> <td>3</td> </tr> <tr> <td>y</td> <td>0</td> <td>1.665</td> <td>3.144</td> </tr> </table> area $\approx \frac{1}{2} \times 1 \times [0 + 3.144 + 2(1.665)] = 3.24$ (3sf)</p> <p>(b) volume = $\pi \int_1^3 x^2 \ln x dx$ $u = \ln x, \quad u' = \frac{1}{x}, \quad v' = x^2, \quad v = \frac{1}{3}x^3$ $I = \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2 dx$ $= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + c$ volume = $\pi [\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3]_1^3$ $= \pi \{(9 \ln 3 - 3) - (0 - \frac{1}{9})\}$ $= \pi(9 \ln 3 - \frac{26}{9})$</p>	x	1	2	3	y	0	1.665	3.144	B1 B1 M1 A1 M1 M1 A2 A1 M1 A1	(11)
x	1	2	3								
y	0	1.665	3.144								

5. (a) $\frac{5-8x}{(1+2x)(1-x)^2} \equiv \frac{A}{1+2x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$
 $5-8x \equiv A(1-x)^2 + B(1+2x)(1-x) + C(1+2x)(1-x)$ M1
 $x = -\frac{1}{2} \Rightarrow 9 = \frac{9}{4}A \Rightarrow A = 4$ A1
 $x = 1 \Rightarrow -3 = 3C \Rightarrow C = -1$ A1
 coeffs $x^2 \Rightarrow 0 = A - 2B \Rightarrow B = 2$ M1 A1
 $f(x) = \frac{4}{1+2x} + \frac{2}{1-x} - \frac{1}{(1-x)^2}$
- (b) $f(x) = 4(1+2x)^{-1} + 2(1-x)^{-1} - (1-x)^{-2}$
 $(1+2x)^{-1} = 1 + (-1)(2x) + \frac{(-1)(-2)}{2}(2x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2}(2x)^3 + \dots$ M1
 $= 1 - 2x + 4x^2 - 8x^3 + \dots$ A1
 $(1-x)^{-1} = 1 + x + x^2 + x^3 + \dots$ B1
 $(1-x)^{-2} = 1 + (-2)(-x) + \frac{(-2)(-3)}{2}(-x)^2 + \frac{(-2)(-3)(-4)}{3 \times 2}(-x)^3 + \dots$
 $= 1 + 2x + 3x^2 + 4x^3 + \dots$ A1
 $f(x) = 4(1 - 2x + 4x^2 - 8x^3) + 2(1 + x + x^2 + x^3) - (1 + 2x + 3x^2 + 4x^3)$ M1
 $= 5 - 8x + 15x^2 - 34x^3 + \dots$ A1
- (c) $|x| < \frac{1}{2}$ A1 (12)

6. (a) $\frac{dx}{dt} = 1 + \cos t, \quad \frac{dy}{dt} = \cos t$ M1
 $\frac{dy}{dx} = \frac{\cos t}{1 + \cos t}$ M1 A1
- (b) $\frac{\cos t}{1 + \cos t} = 0, \quad \cos t = 0, \quad t = \frac{\pi}{2}$ M1 A1
 $\therefore (\frac{\pi}{2} + 1, 1)$ A1
- (c) $= \int_0^{\pi} \sin t \times (1 + \cos t) dt = \int_0^{\pi} (\sin t + \frac{1}{2} \sin 2t) dt$ M1 A1
 $= [-\cos t - \frac{1}{4} \cos 2t]_0^{\pi}$ M1 A1
 $= (1 - \frac{1}{4}) - (-1 - \frac{1}{4}) = 2$ M1 A1 (12)

7. (a) $\overrightarrow{AB} = (8\mathbf{j} - 6\mathbf{k}) - (3\mathbf{i} + 6\mathbf{j} - 8\mathbf{k}) = (-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ M1
 $\therefore \mathbf{r} = (3\mathbf{i} + 6\mathbf{j} - 8\mathbf{k}) + \lambda(-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ A1
- (b) $3 - 3\lambda = -2 + 7\mu \quad (1)$
 $6 + 2\lambda = 10 - 4\mu \quad (2)$
 $-8 + 2\lambda = 6 + 6\mu \quad (3)$ B1
 $(3) - (2): -14 = -4 + 10\mu, \mu = -1, \lambda = 4$ M1 A1
 check (1) $3 - 12 = -2 - 7$, true \therefore intersect B1
- (c) $\mathbf{r} = (-2\mathbf{i} + 10\mathbf{j} + 6\mathbf{k}) - (7\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}) \quad \therefore (-9, 14, 0)$ M1 A1
- (d) $\overrightarrow{OC} = [(-2 + 7\mu)\mathbf{i} + (10 - 4\mu)\mathbf{j} + (6 + 6\mu)\mathbf{k}]$
 $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = [(-5 + 7\mu)\mathbf{i} + (4 - 4\mu)\mathbf{j} + (14 + 6\mu)\mathbf{k}]$ M1 A1
 $\therefore [(-5 + 7\mu)\mathbf{i} + (4 - 4\mu)\mathbf{j} + (14 + 6\mu)\mathbf{k}] \cdot (-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) = 0$ M1
 $15 - 21\mu + 8 - 8\mu + 28 + 12\mu = 0$ A1
 $\mu = 3 \quad \therefore \overrightarrow{OC} = (19\mathbf{i} - 2\mathbf{j} + 24\mathbf{k})$ M1 A1 (14)

Total (75)

